Modeling of the Scale of Separations in Cockpit Displays for Limited Delegation of Separation Assurance

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ABSTRACT

The Evolutionary Air-ground Co-operative ATM Concepts (EACAC) study of the Freer-Flight project investigates the delegation by the controller to the pilot, of some tasks related to separation assurance. Starting from the analogy of visual clearances, EACAC investigates the possibility of giving electronic clearances. One of the issues of the study deals with the appropriate assistance scheme to be provided to the pilot, considering the proposed concept of limited delegation. The “scales of separations” is one of the levels of assistance envisaged. Two models of turns and their appropriateness for lateral scales of effects are investigated. The scales of separations implementing these models will be evaluated by pilots, first through a cockpit environment in a stand-alone mode (end 1999), and then using a cockpit simulator in a real-time simulation with controllers (mid 2000).

INTRODUCTION

The Evolutionary Air-ground Co-operative ATM Concepts (EACAC) study of the Freer-Flight project investigates the delegation by the controller to the pilot, of some tasks related to separation assurance. Starting from the analogy of visual clearances, EACAC investigates the possibility of giving electronic clearances by making use of the new CNS/ATM technologies soon to be available, in conjunction with new operational procedures. EACAC targets near term applications – typically 2005 – taking place in current ATC organizations, while at the same time proposing long-term developments. Expected gains are mainly a reduction of controller workload together with the improvement of safety and flight efficiency.

One of the issues investigated by the study is the appropriate assistance scheme to be provided to the pilot, considering the proposed concept of limited delegation. An initial design work has been done, leading to the definition of key elements for cockpit displays: three different levels of assistance are identified, with different representations. One of them is referred to as “scales of separations”. The objective of this paper is to investigate the intrinsic characteristics of the scales of separations in the case of a lateral maneuver and to discuss a way to compute them considering aircraft kinematics.

The paper is organized as follows: the following section outlines the concept of limited delegation. The next section presents an overview of existing and enhanced CDTI. In the final section, two models of turn and their appropriateness for lateral scales of separations are investigated.

OUTLINE OF THE CONCEPT

Two major constraints are driving the EACAC study. The first one relates to human acceptability, and can be expressed by: respect roles and working methods of controllers and pilots, enable incremental practice and progressive confidence. Beyond, the underlying issue is to overcome the acceptance dilemma: no human-centred system can be used without confidence, but no confidence can be obtained without practising the system in real operations. The second constraint relates to technology: rely on minimum assumptions for CNS facilities and equipment modification. In addition, to enable transition phases compatibility with stepwise fleet equipment should be guaranteed.

To meet these constraints, EACAC relies on a pragmatic and straightforward initial concept. Firstly, as for visual clearances, the concept is applicable for problems involving two aircraft: one aircraft (subject aircraft) receives the delegation with respect to the other aircraft (target aircraft). In addition, depending on the task delegated, conditions of applicability in terms of complexity of problems

have been identified, and must be respected. Secondly, the concept relies on the following key points:

- **Limited delegation**: The task delegated to the pilot is limited to the monitoring and implementation of solutions. Thus, situation analysis, identification of problems, e.g., conflict detection, definition of solutions, and decision of transfer remain within role and responsibility of the controller. (However, the delegation requires pilot's agreement.)

- **Flexible use of delegation**: The level of task delegated to the pilot can range from monitoring up to implementation of a solution, and the controller has the ability and responsibility to select for each problem the appropriate level of delegation.

EACAC aims at covering two major classes of application: crossing and overtaking, typically in en-route airspace, and sequencing, typically in TMA (Terminal Maneuvering Area). For each of these applications, three levels of delegation have been identified: basic, intermediate, and advanced. For the basic level, the delegation is extremely limited, hence enabling early practice. Beyond, the higher the delegation level, the higher the expected gain. However, a higher level imposes more restrictive conditions of applicability.

For crossing and overtaking applications, the three following levels of delegation have been identified:

1. **Basic**: identification of the "clear of traffic".
2. **Intermediate**: climb/descent or navigation resume.
3. **Advanced**: implementation of maneuver.

For sequencing applications, the three following levels of delegation have been identified:

1. **Basic**: identification of separation.
2. **Intermediate**: station keeping.
3. **Advanced**: sequence establishment and traffic merging.

In terms of surveillance information, position and velocity of the target aircraft transmitted through ADS-B or TIS-B are sufficient. Trajectory intent information (such as Trajectory Change Points) is not required since identification of problems and solutions is performed by the controller, but can be used to increase applicability domain. In addition, surveillance information on surrounding traffic is not required.

**OVERVIEW OF EXISTING COCKPIT DISPLAYS OF TRAFFIC INFORMATION**

A wide range of applications along with the appropriate on-board assistance scheme, has been investigated so far in the domain of air/ground co-operative ATM. Typically, applications investigated range from short term, e.g., station keeping and in-trail following, to longer term, e.g., autonomous aircraft under free-flight operations. A possible way to classify and characterize the various on-board assistance schemes thus defined is to consider the level of assistance provided. However, since some are used in a context of autonomous aircraft operations, they integrate not only "resolution" features, but also "alerting" ones [13]. In the context of limited delegation where the controller always performs conflict detection, only the "resolution" aspect will be discussed. Three major levels of assistance can be identified:

- **"Actual"**: presents the situation based on current flight parameters of subject aircraft. It may include some predictive features, typically for displaying the point of loss of separation or the point of closest approach. The display proposed in [3] to support the "ASAS crossing procedure" is a typical example based on flight state information: the situation with respect to the target aircraft is presented through a relative speed line and a 8Nm circle centered on the subject aircraft position. A set of displays using different levels of intent information (flight state, commanded values and trajectory) to indicate conflict bands is proposed and evaluated in [1].

- **"What-if"**: provides the capability to test potential maneuvers. Typically, [10] has defined a predictive tool to assess a risk of conflict, which uses target values selected on the auto-pilot. The effect of potential changes of heading and/or speed can thus be tested before they are engaged. For an FMS based approach, [5] provides a graphic editing capability for the subject aircraft trajectory which is used in conjunction with "no-go" zones generated by both conflicting and surrounding aircraft.

- **"Red/green range"**: indicates the range of authorized and forbidden maneuvers. The example is TCAS, which provides red and green arcs on the vertical speed indicator. (For heading or bank angle with the former TCASIII, similar principles were envisaged.)

- **"Advisory"**: indicates the maneuver to be performed. Again, TCAS is a typical example of a system providing advisories, as well as the system proposed by [8]. The pilot has to follow exactly the maneuver computed to solve the conflict (or to avoid the collision for TCAS). The solution trajectory automatically generated in [5] can also be seen as an advisory. In that case however, the pilot can ask for other solution trajectories by giving high level directives, e.g., pass to the left.

As stressed in [11], the on-board assistance scheme required is closely linked to the underlying operational concept and related procedures, e.g., what tasks are intended to be delegated to the pilot, and how? Typically, the level "actual" should be sufficient for a monitoring task, whereas a "what-if" should be the minimum level of assistance for implementing a maneuver. However, finding the appropriate maneuver with a "what-if" requires varying one (and possibly more) flight parameter. The

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2. Regardless that TCAS is designed for collision avoidance, and not for separation assurance.
some preventive indications could be introduced after “clear of traffic”. Instead, a “what-if” naturally provides a simple way to do it: the “resume” maneuver can be tested before it is actually engaged.

Considering the possible limitations of the “what-if” on one side (multiple testing), and the “advisory” on the other (mainly automation dependency), an intermediate assistance level has been envisaged: the scale of separations that indicates the separation value for a range of possible maneuvers. The “scale of separation” can be seen as an extension of the binary “red/green ranges” towards an “analog” representation of the effect of different maneuvers. In addition, it remains valid and meaningful even after “clear of traffic”.

REQUIREMENTS FOR AN ENHANCED COCKPIT DISPLAY OF TRAFFIC INFORMATION

In the scope of identifying the main elements for an enhanced CDTI, different aspects are considered. The first two ones lie on the functions required along with the level of assistance. The other aspects are the type of representation, the level of information used, and the definition of the separation parameter.

To support operations of limited delegation as presented before, two assistance functions are required:

- Monitoring separations, including identification of specific events typically the “clear of traffic”.
- Implementation of solutions, i.e. capability to identify of an appropriate maneuver (provided that the type of maneuver has been specified by the controller).

For that purpose, the three following levels of assistance are proposed:

- “Actual”*: indicates the separation value based on current flight parameters.
- “Target/what-if”*: indicates the separation value based on the target values of flight parameters selected on the auto-pilot (heading, speed, or vertical speed). As a consequence, this also provides “what-if” assistance if a target flight parameter is in the HOLD mode of the auto-pilot (applicable for heading and speed).
- “Scale of separations”: indicates the separation value for a range of flight parameters (heading or speed). Knowing the desired separation, an advisory can easily be obtained.

Typically, it is thought that monitoring separations will require the first and possibly the second assistance level, while the implementation of solutions will probably require the second or the third.

The information used on the subject aircraft is the flight states and the target values of the auto-pilot. On the target aircraft only flight states information is required, though intent can be used if available.

The definition of the separation parameter depends on the application. It is based on:

- Closest point of approach (CPA) for high closure rate applications, e.g. lateral and vertical crossing.
- Sliding future point (SFP), i.e. predicted point x minutes in advance, for low closure rate applications, e.g. lateral and vertical low convergence and passing, longitudinal station keeping.

LATERAL MANEUVER MODELING

The focus of the work here is the modeling of a lateral maneuver (i.e. a turn) to maintain the separation with another aircraft, as done in [2]. Two models of an autopilot constant bank angle turn are compared to recordings performed on full 6 degree of freedom Boeing 747 flight simulator. The objective is to find the simplest yet accurate algorithm for predicting the distance at the closest point of approach as a function of track change.

By convention, in this paper, ‘1’ denotes the maneuvering aircraft (own aircraft) while ‘0’ denotes the target aircraft. Aircraft 0 is assumed to be flying at constant speed along a straight line. Let \( n \in \{0,1\} \) be the index referring to the aircraft, \( (v_n, y_n) \) the positions at a given time, \( (x^0_n, y^0_n) \) the initial position, \( y_n \) the magnitude of the speed (assumed constant for the duration of the maneuver), \( \psi_n \) the track angle and \( \mu \) the bank angle in turn.

INSTANTANEOUS TURN – This is the most basic model: the track change is assumed to be instantaneous, yielding a ‘corner’ in the trajectory. Since simplicity is implicit, the aim is to find the limitations of such a model for separation purposes.

Assuming a turn at \( t = 0 \), the trajectory of each aircraft is simply:

\[
\begin{align*}
\dot{y}_n &= \psi_n \mu_n \\
\dot{x}_n &= \frac{\psi_n}{\cos \psi_n} \mu_n
\end{align*}
\]

3. In a typical use however, only one flight parameter will be modified.
Then, the distance between the two aircraft is:

\[ d = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2} \]

The time and the distance at the point of closest approach are obtained by taking the derivative with respect to time of the distance above.

Assuming that:

\[ (v_0 \cdot \cos(\psi_0) - v_1 \cdot \cos(\psi_1))^2 + (v_0 \cdot \sin(\psi_0) - v_1 \cdot \sin(\psi_1))^2 \neq 0 \]

Then:

\[ t_{min} = \frac{\left[ (x_1^0 - x_0^0) \cdot (v_1 \cdot \cos(\psi_1) - v_0 \cdot \cos(\psi_0)) + (y_1^0 - y_0^0) \cdot (v_1 \cdot \sin(\psi_1) - v_0 \cdot \sin(\psi_0)) \right]}{\left( v_1 \cdot \cos(\psi_1) - v_0 \cdot \cos(\psi_0) \right)^2 + \left( v_1 \cdot \sin(\psi_1) - v_0 \cdot \sin(\psi_0) \right)^2} \]

Finally, two distinct cases must be considered:

- if \( t_{min} \) is negative or undefined, meaning that the two trajectories are either diverging or parallel, then:

\[ d_{min} = \sqrt{(x_1^0 - x_0^0)^2 + (y_1^0 - y_0^0)^2} \]

- or

\[ d_{min} = \sqrt{\left[ \frac{x_1^0 - x_0^0 - t_{min} \cdot (v_1 \cdot \cos(\psi_1) - v_0 \cdot \cos(\psi_0))}{v_1 \cdot \cos(\psi_1) - v_0 \cdot \cos(\psi_0)} \right]^2 + \left[ \frac{y_1^0 - y_0^0 - t_{min} \cdot (v_1 \cdot \sin(\psi_1) - v_0 \cdot \sin(\psi_0))}{v_1 \cdot \sin(\psi_1) - v_0 \cdot \sin(\psi_0)} \right]^2} \]

**INSTANTANEOUS BANK ANGLE** – In the second model, the bank angle change (instead of the track angle) is assumed to be instantaneous, hence providing more realism and accuracy than the first model. The maneuvering aircraft trajectory is composed of two successive segments:

1. an arc circle modeling the turn
2. a straight-line once the turn is completed.

During the first phase, the maneuvering aircraft turns with a constant bank angle. The variables at the end of this phase are denoted by \( \bullet ^1 \). The balance of forces during a sustained coordinated turn yields, with \( g \) being the gravity:

\[ v_1 \psi_1 = \tan(\mu) g \]

Therefore, the angular speed is constant:

\[ \psi_1 = \frac{\tan(\mu) g}{v_1} \]

and the turn lasts:

\[ t^1 = \frac{\psi_1}{\psi_1} \]

Then, the equations of the trajectory for Aircraft 1 during the first phase are:

\[ \begin{align*}
    x_1 &= x_1^0 + v_1 \sin(\psi_1 t) \\
    y_1 &= y_1^0 + v_1 (1 - \cos(\psi_1 t))
\end{align*} \]

until \( t^1 \).

The trajectory of Aircraft 1 is an arc circle while the other aircraft’s is a straight line. The distance between the two aircraft is defined with the same formula as for the first model. Because of time dependency in the trigonometric terms, a purely numerical method is used to find \( t_{min}^1 \) and \( d_{min}^1 \).

The equations describing the second phase are those of the first model investigated except for the initial conditions.

\[ \begin{align*}
    x_1^1 &= x_1^0 + \frac{v_1}{\psi_1} \sin(\psi_1) \\
    y_1^1 &= y_1^0 + \frac{v_1}{\psi_1} (1 - \cos(\psi_1)) \\
    x_1 &= x_1^1 + v_1 \cdot \cos(\psi_1) \cdot (t - t^1) \\
    y_1 &= y_1^1 + v_1 \cdot \sin(\psi_1) \cdot (t - t^1)
\end{align*} \]

The minimal separation for the second phase (called \( d_{min}^2 \)) can be found by the same way as for the first model. The results are also similar:

- if \( t_{min}^2 \) is smaller than \( t^1 \) or undefined, meaning that the two trajectories are respectively diverging or parallel, then:

\[ d_{min}^2 = d^1 \]

- or

\[ d_{min}^2 = \sqrt{\left[ \frac{\frac{x_1^0 - x_0^0 - t_{min}^2 \cdot (v_1 \cdot \cos(\psi_1) - v_0 \cdot \cos(\psi_0))}{v_1 \cdot \cos(\psi_1) - v_0 \cdot \cos(\psi_0)} \right]^2 + \left[ \frac{\frac{y_1^0 - y_0^0 - t_{min}^2 \cdot (v_1 \cdot \sin(\psi_1) - v_0 \cdot \sin(\psi_0))}{v_1 \cdot \sin(\psi_1) - v_0 \cdot \sin(\psi_0)} \right]^2} \]

The overall minimum distance between the two aircraft is straightforwardly:

\[ d_{min} = \min\{d_{min}^1, d_{min}^2\} \]
EVALUATION OF THE MODELS

Position error after a turn – In this section, the trajectories obtained using either of the two models introduced are compared with trajectories recorded from the Boeing 747 flight simulator using full 6 degree of freedom dynamics as well as accurate aerodynamic and auto pilot models. Figure 1 depicts a plan view of the 3 trajectories for a flight at FL330 and 465 knots.

For the following turn angles (10°-30°-50°-90°), the maximum distance between the reference trajectory and the model up to the end of the turn are evaluated. This is intended to provide an upper bound to the error when estimating the distance at the closest point of approach using these models. Note that the projection from the latitude / longitude coordinate system used by the simulator into Cartesian coordinates produce some minor artifacts: the constant heading trajectories are not perfectly straight lines. But these discrepancies were found orders of magnitude smaller that the error introduced by the models hence negligible.

Figure 2 shows the evolution of the error as a function of the track angle change for the two models. It shows quite clearly that the errors are monotonically increasing with the track angle change. Keeping in mind that the required separation distance may typically be in the order of 5 to 8 NM, the error with the instantaneous turn model is already in the order of 0.5 NM for a 15 degree track change, suggesting a practical limit of about 10 degree for this model. With the instantaneous bank assumption the 0.5 NM error is reached for track changes above 50 degrees. Modeling the time it takes to bank and the resulting transient in turn rate would further more reduce this error.

Altitude Dependency – There is no explicit dependence on altitude in either of the two models. Comparison performed using reference trajectories recorded at various altitudes shows that as long as the nominal turn bank angle is not altered, the differences between 2 reference trajectories at the same true airspeed but at different altitude never exceed 0.06 NM. The difference can be attributed to the variation in the time required to bank the aircraft.

Speed Dependency – The dependency in speed at a given altitude (FL330) for the first model is shown in Table 1.

Table 1. Speed dependency, instantaneous turn assumption

<table>
<thead>
<tr>
<th>Speed (knots)</th>
<th>413</th>
<th>465</th>
<th>528</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error 10° (NM)</td>
<td>0.26</td>
<td>0.28</td>
<td>0.31</td>
</tr>
<tr>
<td>Error 30° (NM)</td>
<td>1.02</td>
<td>1.30</td>
<td>1.60</td>
</tr>
</tbody>
</table>

The error increases with speed. This can be traced to the fact that for the reference trajectory the higher the speed the larger the turn radius and hence the error due to ignoring the turn phase.

For the second model, a similar dependency can be observed between the speed and the error, as shown in Table 2. However, a sharper increase of the error occurs for upper speed range. This can be traced to a bank angle limitation by the auto-pilot for g-load protection. At 528 knots, a maximum of 23 instead of the nominal 25 degrees of bank is used in the simulator. It highlights that the accuracy of second model is very dependent on the knowledge of the bank angle (or turn radius) that is being used at this point of the flight envelope by the auto-pilot.
EVOLUTION OF THE INDICATED TRACK – The models developed previously are now used to assist the flight crew in maintaining a given separation standard. It is assumed that, at any given time, the pilot is shown on a display the estimation of the track to follow to obtain the preset separation value. Figure 3 shows a possible implementation (see [14] for overall principles).

At $t = 0$, the pilot selects the track $\psi$ matching the desired separation value $d_{SEP}$ through the auto-pilot: $\psi^0_{d_{sep}}$. Because of the errors of the models, the track satisfying the separation standard $\psi_{d_{sep}}$ will vary with time. The aim of this section is to compute this evolution and analyze it in particular with respect to the stability of the indications.

A reference trajectory, including at $t = 0$ the start of a known track change $\psi^0_{d_{sep}}$, is adjusted laterally so as to produce the desired separation $d_{SEP}$ (in general 8 NM) at the point of closest approach and after the completion of the track change. Because of the modeling errors, the separation standard corresponding to $\psi^0_{d_{sep}}$, $d_{SEP}$ differs from $d_{SEP}$. Then, $\psi_{d_{sep}}$ is computed all along the trajectory.

The experiment has been performed for the two models and for two track angles, 10 and 50 degrees, for a resulting minimal separation $d_{SEP}$ of 8 NM and an overall flight time between the start of turn and point of closest of approach in the 1 to 2 minutes range.

10° turn – The objective of this first experiment is to evaluate the first model on small angles. The desired separation $d_{SEP}$ is assumed to be 8.2 NM although the achieved separation $\bar{d}_{SEP}$ is designed to be 8 NM. Figure 4 represents the indicated track $\psi_{d_{sep}}$ as a function of time.

At $t = 0$, the indicated track $\psi_{d_{sep}}$ for $d_{SEP} = 8.2$ NM is 10 degrees, hence the 10 degree track change initiated by the flight crew.

During the first few seconds, $\psi_{d_{sep}}$ continues to increase as before the turn initiation. This is due to time required to bank the aircraft and actually start turning. Then, the indication is almost constant, slightly drifting at a rate of 0.015 degree/s, for the next 80 seconds. $\psi_{d_{sep}}$ stays within 2 degrees of the initially chosen track $\psi^0_{d_{sep}}$. The last part of the graph illustrates that near the CPA, the indicated track $\psi_{d_{sep}}$ diverges. Because of the difference between the desired separation $d_{SEP}$ (8.2 NM) and the one achieved $\bar{d}_{SEP}$ (8 NM), when the aircraft reaches the desired minimum separation distance $d_{SEP}$, a singularity occurs when trying to compute $\psi_{d_{sep}}$. From this point on and beyond, the desired minimum separation has been infringed and no aircraft track value can prevent it anymore. Note that if $d_{SEP} \geq d$, no track angle $\psi_{d_{sep}}$ can be obtained.

With the second model, the precision is better ($|\bar{d}_{SEP} - d_{SEP}| = 0.12$ NM instead of 0.2 NM). The evolution of $\psi_{d_{sep}}$ is identical. The same divergence occurs near the desired minimum separation.

<table>
<thead>
<tr>
<th>Speed (knots)</th>
<th>413</th>
<th>465</th>
<th>528</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error 30° (NM)</td>
<td>0.30</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>Error 90° (NM)</td>
<td>0.65</td>
<td>1.77</td>
<td></td>
</tr>
</tbody>
</table>
In this case, there is no significant difference between the two models. However, it already appears very clearly that an indicated track angle for a given separation value becomes very unstable in the aircraft distance is in the vicinity of the separation value. Such an indication is likely not sufficient to monitor the separation distance and it must be augmented with at least the separation associated with the current track and possibly with a full scales of separations.

50° turn – It is already known that for such an angle, the first model is too simple to give good results. Nevertheless, the pilot could have the option to iteratively increase the angle of turn during the maneuver. However, the indicated track $\psi_{d_{sep}}$ must be sufficiently stable for that to be feasible. Figure 5 represents the indicated track $\psi_{d_{sep}}$ as a function of time when the first model is used.

The error in prediction $|\ddot{d}_{sep} - d_{sep}|$ is very large: 2.37 NM if no correction is performed after the initiation of track change. This is not acceptable. Moreover, the indicated track $\psi_{d_{sep}}$ never stabilizes during the maneuver making manual adjustment to the selected track potentially difficult to implement for the flight crew. This excludes the use of the first model when any significant heading change (about 15 degrees) is required.

With the second model, the precision $|\ddot{d}_{sep} - d_{sep}|$ is better with a value of 0.34 NM. This is to compare with the desired separation of 8 NM, hence an error of 4.25%. Figure 6 represents the indicated track $\psi_{d_{sep}}$ as a function of time when the first model is used.

At the beginning of the turn, the indicated track $\psi_{d_{sep}}$ continues to increase due to the latency of the turn initiation. This is the same behavior as the one observed for the 10 degree track change using either model. Then, the indicated track stabilizes at about 54 degrees for the next 40 seconds. As a matter of fact, this stable zone corresponds to the actual turn phase for the two trajectories. The second model is rather accurate during the turn, but does not explicitly model the entry and exit of turn. Therefore, as the error essentially due to the turn entry transient is already detected during the stable segment, the flight crew can easily perform a correction during the turn.

Figure 7 illustrates the potential benefit of such a correction. In this case, it is assumed that 20 seconds after the turn initiation, the flight crew updates the auto-pilot target track to the current indicated track $\psi_{d_{sep}}$. Therefore the turn initiation transient error is cancelled with only the turn exit transient error remaining.

From this example, it appears that the second model is appropriate even for large track angle changes. Furthermore, a track angle adjustment can easily be performed to improve the accuracy if necessary. It finally indicates that it is not necessary to explicitly model the bank angle transient with its intrinsic added complexity.

SEPARATION DISTANCE TO TRACK ANGLE CHANGE INVERSION – In the previous section, the accuracy of the two models and their applicability are investigated. However, earlier in the derivation of the equations, the separation distance $d_{sep}$ is computed as a function of the track angle change $\psi_{d_{sep}}$. But, for display reasons as shown in Figure 3, finding the track angle change corresponding to a given separation distance is more relevant. Therefore, the equations have to be inverted. Because of the complexity of the expressions, a numerical inversion rather than an analytical one is proposed.
However, it must be noted that inversion algorithms have often requirements in terms of monotonousness of the subject function. The following gives an example of an aircraft crossing geometrical configuration and the consequence in terms of $d_{SEP}$ as a function of $\psi_{d_{SEP}}$. This is the result of a long but non exhaustive and non scientific search process to highlight configurations that may cause difficulties.

The selected case is a slow overtaking configuration. The initial conditions for the target aircraft are $x_0^0 = 0$, $y_0^0 = 0$, $\psi_0 = 0$ and $v_0 = 450$ knots and as well as for the own: $x_1^0 = -60$ NM, $y_1^0 = -60$ NM, $\psi_1^0 = 5$ degrees and $v_1 = 500$ knots.

Figure 8 shows the time and distance at the closest point of approach for the complete spectrum of heading changes.

![Figure 8. Time and distance at CPA for an overtaking configuration](image)

The distance curve has multiple peaks hence is non monotonous and will break simple search algorithms like the one based on dichotomy or on a gradient approach. Figure 8 also highlights the importance of considering simultaneously the time at CPA: indeed in this configuration, at a zero degree track angle change, the aircraft has to fly more than one hour before reaching the CPA. Furthermore the predicted distance at the CPA is subject to significant uncertainty if relying purely on flight track extrapolation: flying at a speed of 450 knots, an error of 0.1 degree in track angle yields a 0.8 NM position error for a 1 hour extrapolation.

Furthermore, Figure 8 shows, in the neighborhood of a -90 track angle change, a typical minimum in separation distance which might be overlooked by a coarse algorithm. Finally, these time and distance at CPA indications must be used with careful judgment: in the example given, a left turn may appear as a good solution on the graph (a 15 degree turn, yields a minimum separation of 85 NM reached in 12 s). But it will simply put both aircraft on parallel / diverging trajectories and the separation problem will reappear as soon as the aircraft will try to head again towards its destination. Alternatively, a right turn can allow the aircraft to fly behind its target while maintaining the separation and afterwards safely head back towards its destination. Multiple track changes may yield the same separation distance. So providing the complete information on the separation distance achieved depending on the track change will allow the flight crew to decide which track change is the most appropriate from an operational point of view.

**CONCLUSION**

In the scope of designing on-board assistance schemes for limited delegation of separation assurance, scales of separations is one of the levels of assistance envisaged in cockpit displays. Two possible models for the implementation of track change scales of separations have been proposed. Their characteristics and their limitations have been investigated. A simple turn model with instantaneous bank angle change is shown to be adequate for separation assurance purposes. It is applied to an overtaking scenario to highlight potential numerical and operational issues. The scales of separations implementing these models will be evaluated by pilots, first through a cockpit environment in a stand-alone mode (end 1999), and then using a cockpit simulator in a real-time simulation with controllers (mid 2000).

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